Computers and Chemical Engineering, Vol. 2, pp. 19–32
© Pergamon Press Ltd., 1978. Printed in Great Britain

0098-1354/78/0301-0019/$02.00/0

NUMERICAL AND EXPERIMENTAL SIMULATION STUDIES ON THE MIXING OF PARTICULATE SOLIDS AND THE SYNTHESIS OF A MIXING SYSTEM

MIXING PROCESS AND STOCHASTIC MOTION OF MUTUALLY NONINTERACTING PARTICLES

L. T. FAN*
Department of Chemical Engineering, Kansas State University, Manhattan, KS 66506, U.S.A.

F. S. LAI†
U.S. Grain Marketing Research Center, ARS, USDA, 1515 College Avenue, Manhattan, KS 66502, U.S.A.

and

Y. AKAO, K. SHINODA and E. YOSHIZAWA
Department of Computer Science, Yamanashi University, Kohu, Yamanashi 500, Japan

(*Revised* 15 *September* 1977)

Abstract—Mixing in a conventional solids mixer is governed mainly by two basic mechanisms, diffusion and convection. The characteristics of these processes have been investigated separately. Model mixers were constructed using these processes, and a new mixing system was synthesized by a proper combination of the two. Performances of the elementary model mixers, as well as the synthesized mixing system, were simulated on the computer, and the results of simulation were experimentally verified. The method of synthesis, which appears to be effective, can be extended to the design of other practical mixing systems.

Scope—Mixing is an operation of mingling different particle groups by imparting to them complicated motions by mechanical means. This simultaneously produces a fairly regular deterministic bulk flow of particles and very irregular stochastic movement of the individual particles. It is extremely difficult, if not impossible, to describe such a phenomenon by means of the classical deterministic mechanics. The use of probabilistic approaches in this field is still in its infancy. Furthermore, empiricism has played a dominant role. Significant advances may never be attained if we continue to employ purely empirical and classical approaches.

Conventionally, blending or mixing of solid particles has been carried out by moving, rotating, and/or vibrating containers, by rotating blending blades, and by passing the particles through specially designed containers. Lacey[1] indicated that the mechanical mixing processes induced by such complicated motions can be classified into three elementary mechanisms: diffusive mixing, convective mixing and shear mixing. The third mechanism, however, can be considered as a combination of the first two mechanisms occurring simultaneously[2].

The objective of this paper is to develop a new and fundamental approach to the analysis of a mixing operation and to the synthesis of a system for mixing two different types of particulate solids.

Conclusions and Significance—Conventionally, solids mixing has been regarded as a simultaneous process involving both convective and diffusive mixing. The need and possibility of investigating these two elementary mechanisms separately for the purpose of attaining a deeper insight into the process are emphasized.

The branching model is shown to be representative of the diffusive mechanism. Each particle falling through the inclined board of the mixer with rows of hexagonal blocks has a prior probability of 1/2. The stratified feeding model is shown to be representative of the convective mechanism.

It is shown that the new mixing system can be synthesized by a combination of the two elementary mixing processes. Performances of the elementary model mixer as well as the synthesized new mixing system were simulated on a computer and the results of simulations were experimentally verified. The synthesized mixing system appears to be effective. The proposed method of synthesis can be extended to the design of other practical mixing systems.

1. INTRODUCTION

The design of mixers for particulate solids has been based mainly on experience. The fundamental theories underlying mixing operations have not been firmly established. Mixing applications are becoming more numerous in today's industries. Very often each industry has developed mixers for its own use, which has led to a wide diversification of the design of mixers and blenders. Notwithstanding the contributions by several pioneers[1, 3–6], progress in the field of solids mixing has been slow. This may be due to the inherently complex nature of mixing processes and the many uncertainties involved in practical mixing operations.

*Correspondence should be addressed to this author.
†Formerly, Graduate Research Assistant, Department of Chemical Engineering, Kansas State University, Manhattan, KS 66506, U.S.A.

Mixing is an operation of mingling different particle groups by imparting to them complicated motions by mechanical means. This simultaneously produces fairly regular deterministic bulk flow of particles and very irregular stochastic movement of individual particles. It is extremely difficult, if not impossible, to describe such a phenomenon by means of Newtonian mechanics. The use of probabilistic approaches in this field is still in its infancy. Furthermore, empiricism has played a dominant role. Significant advances may never be attained if we continue to employ purely empirical and conventional non-unified approaches.

The objective of this work is to develop a new and fundamental approach to the analysis of a mixing operation and to the synthesis of a mixing system. Mixing in a conventional mixer is governed mainly by two basic mechanisms, diffusion and convection. A mixing process depending solely on one mechanism will be called an elementary mixing process. Two elementary mixing processes, one depending on diffusion only and the other on convection only, will be investigated separately. The basic characteristics of these processes will be discussed, with an attempt to show how a new mixing system can be synthesized through their proper combination. Finally, the new system is experimentally verified and numerically simulated.

2. ELEMENTARY MIXING PROCESSES

One of the best understood mixing phenomena is the purely diffusional mixing of gas molecules[7]. However, the mixing behavior of solid particles is generally far more complicated than that of gas molecules. The size of solid particles is usually too large for the theories and methodologies of statistical mechanics to be applicable to them, and the number of solid particles involved in a solids mixing and demixing operation is so large that deterministic approaches of particle dynamics are useless. Moreover, since particles involved in solids mixing processes are always completely disjoint, the law of continuum mechanics is not valid for such processes except for idealized situations. It is therefore desirable to establish mechanistic models which are fundamentally different from those of a gas molecule mixing.

A simple mixer with a poor mixing effect can be analyzed easily, but the result of the analysis will be practically useless. On the other hand, it is extremely difficult to analyze the complex behavior of a practical and efficient mixer by conventional means. Therefore, we need a new approach which, unlike the conventional approaches, is not tied to the analysis and design of a specific class of mixers or blenders. Before initiating our efforts to study mixing processes that involve two or more mechanisms simultaneously, we shall consider each elementary mechanism.

(a) *Diffusive mixing process*

Horizontal rotating drum mixers are probably one of the simplest classes of conventional solid mixers. The mixing mechanism in the axial direction of rotation is known to be diffusive[4]. However, the mixing zone is in the top layer, and the transporting zone beneath it does not directly contribute to the mixing action. The same mixing effect as that in the mixing zone of a rotating drum mixer can be obtained by letting particles pass by gravity over an inclined flat plate. In this paper, we shall consider such an idealized mixer. The mixing mechanism in this idealized mixer can be understood through evaluation of prior probabilistic motion of particles.

The idealized mixer to be considered in this study is the probabilistic branching model as shown in Fig. 1. It is an inclined board with rows of hexagonal blocks 12 mm apart; each row is offset from the one above. Glass balls with a diameter of slightly less than 12 mm (about 10.8–11.2 mm) are dropped upon the board, one at a time, from each position above the top row of hexagonal blocks. The balls are allowed to run down between the hexagonal blocks and are collected in the storage column at the bottom of the board. The distribution of balls in the storage column can be calculated, and a probabilistic model can be derived from the distribution. This idealized mixer will give insight into diffusional processes.

If we let particles fall continuously from the same position at the top of the board, the distribution of particles accumulated at the bottom will be approximately normal. This results because, after N times of branchings, the quantitative relationship in the storage columns will be that of the well-known Pascal triangle. Two different colored particles, which originally occupied left- and right-hand sides of the inclined board, were allowed to fall without interference. This probabilistic branching model is similar to the complete mixing tank model in a fluid flow network. The model for the fluid mixing action in recently developed motionless mixers is also similar to the probabilistic branching model[8, 9].

(b) *Convective mixing processes and stratified feeding*

The formulation of a striated mixture of highly viscous material can be obtained by repetitive convective mixing[10]. Its mechanism has been elucidated by Spencer and Wiley[10], and is depicted in Fig. 2. Mohr *et al.*[11, 12] and Mohr[13] discussed the striated mixture in their studies on mixing in laminar flow systems. However, none of them dealt with the effect of the number of striae on the mixing processes. Two groups of particles with the same quantities and dimensions, one on top of the other, are elongated horizontally until their length becomes twice the original length. The mixture is then vertically cut and one half is placed on top of the other. This creates four striae. By repeating this operation,

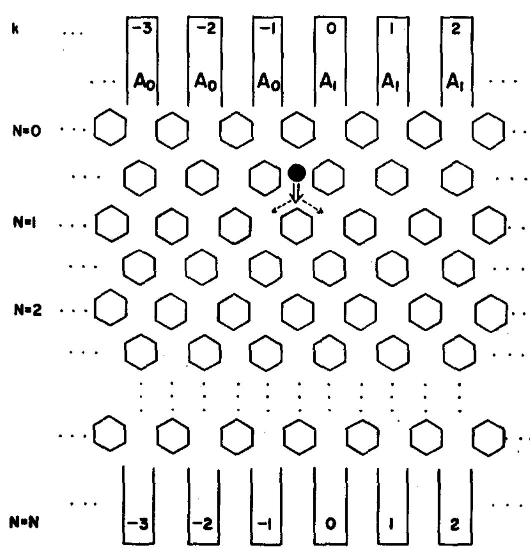

Fig. 1. Probabilistic branching model mixer.

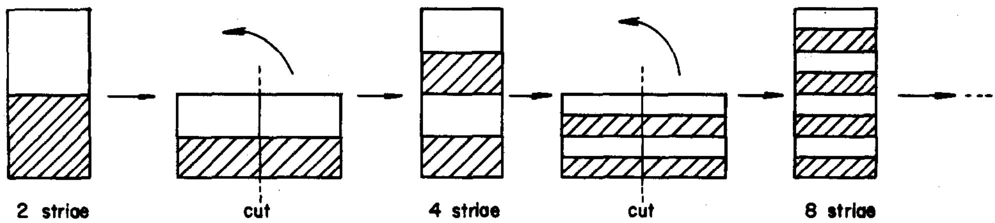

Fig. 2. Formation of a striated mixture.

multi-layer striated mixtures with 8, 16, 32,... striae are obtained. This is essentially convective mixing and is completely different from diffusive mixing. This convective mechanism predominates in pie-kneading operations[14], finger-prone type mixing[15], and recently developed motionless mixers.

The convective mixing mechanism, giving rise to a striated mixture, can be obtained by a proper feeding mode. Such a feeding mode, termed stratified feeding, can be considered a mechanical analog of convective mixing[16]. An increase in the number of separating surfaces of the components increases the total rate of mixing.

Mixing of free falling solid particles in a motionless mixer is mainly affected by two mechanisms: (1) multiple divisions and recombinations of the flow of particles by the helices, i.e. static mixing elements; (2) random interaction of the particles with each other, with the helices, and with the wall of the mixer. By contrast, in the stratified feeding mode, each particle group is subdivided separately, transported and fed stratifiedly. The subdivision and combination of the mixture is mainly done at this stage of the feeding. The subdivision of the particle group occurs only once while the subdivision in motionless mixers occurs repeatedly.

3. DIFFUSIVE MIXING PROCESSES AND NUMERICAL SIMULATION OF A PROBABILISTIC BRANCHING MODEL MIXER

Diffusive mixing processes have been treated most frequently by deterministic diffusional models[2, 4]. On the other hand, Inoue and Yamaguchi[17] have proposed using Markov processes to describe solids mixing in two-dimensional V-type mixers and pan mixers. We shall investigate the diffusive mixing processes with theories of probabilities, Markov processes and diffusion. Specifically, the idealized mixer (Fig. 1) is studied with three different approaches; the coefficients of the binomial series, Markov processes and diffusion theories. By applying these three approaches to the model mixer, we may eventually gain insight into the mechanisms of solids mixing.

(a) *Coefficients of binomial series*

Indices are assigned to the particles to be fed into the probabilistic branching model mixer as shown in Fig. 1. These indices may be called the feeding addresses. Particles of component A_1 are placed at addresses k's ≥ 0, and particles of component A_0 are placed at addresses k's < 0. The concentration distribution of particles of component A_1 among all addresses can be theoretically predicted if each particle is allowed to fall independently from the feeder to stage N. Note that two succeeding rows are counted as one stage, as indicated in Fig. 1. A particle of component A_1 falling from address $k = 0$ at stage $N = 0$ can only go to addresses $k = -1, 0, 1$ at stage $N = 1$.* The proportion of the number of particles in these addresses is ${}_2C_0 : {}_2C_1 : {}_2C_2$, where ${}_mC_n$ is the binomial coefficient representing the number of combinations of m objects taken n at a time if the order of object is unimportant. Each particle of component A_1 falling from any address, $k = 1, 2, \ldots$, will have a similar result. The details of each transition stage are given in Table 1. The number of particles of component A_1 at each address at stage $N = 1$ is:

$$\sum_{m=0}^{k+1} {}_2C_m \quad -1 \leq k \leq 1$$

and the total number of particles in each address is:

$$\sum_{m=0}^{2} {}_2C_m = 2^2.$$

Hence, the concentration distribution at stage $N = 1$ is:

$$\begin{aligned} X_k &= \sum_{m=0}^{k+1} {}_2C_m/2^2 & -1 \leq k \leq 1 \\ &= 0 & k < -1 \\ &= 1 & k > 1. \end{aligned}$$

Similarly, at stage $N = 2$, we have:

$$\begin{aligned} X_k &= \sum_{m=0}^{k+2} \frac{{}_{2\times 2}C_m}{2^{2\times 2}} & -2 \leq k \leq 2 \\ &= 0 & k < -2 \\ &= 1 & k > 2. \end{aligned}$$

In general, at stage N, we have:

$$\begin{aligned} X_k &= \sum_{m=0}^{k+N} \frac{{}_{2N}C_m}{2^{2N}} & -N \leq k \leq N \\ &= 0 & k < -N \\ &= 1 & k > N. \end{aligned} \tag{1}$$

Let us define the total number of particles of component A_1 that have diffused to the left of address $k = 0$ at stage N as M_N; then:

$$M_1 = \sum_{m=0}^{0} {}_2C_m \quad \text{at} \quad N = 1$$

$$M_2 = \sum_{k=-2}^{-1} \sum_{m=0}^{k+N} {}_{2N}C_m$$

*This assumption is equivalent to the assumption of the Markov process in that other states are eliminated.

Table 1. Transition of particles of component A_1 in probabilistic branching model

Component	k	...	-4	A_0 -3	-2	-1	0	1	A_1 2	3	4	...
1 No. of particles of component A_1 from	$k=0$					${}_2C_0$	${}_2C_1$	${}_2C_2$				
	$k=1$						${}_2C_0$	${}_2C_1$	${}_2C_2$			
	$k=2$							${}_2C_0$	${}_2C_1$	${}_2C_2$		
	$k=3$								${}_2C_0$	${}_2C_1$	${}_2C_2$	...
Total						${}_2C_0$	$\sum_{m=0}^{1} {}_2C_m$	$\sum_{m=0}^{2} {}_2C_m$	$\sum_{m=0}^{2} {}_2C_m$	$\sum_{m=0}^{2} {}_2C_m$	$\sum_{m=0}^{2} {}_2C_m$	...
	X_k	...	0	0	0	0.25	0.75	1	1	1	1	...
2 No. of particles of component A_1 from	$k=0$				${}_4C_0$	${}_4C_1$	${}_4C_2$	${}_4C_3$	${}_4C_4$			
	$k=1$					${}_4C_0$	${}_4C_1$	${}_4C_2$	${}_4C_3$	${}_4C_4$		
	$k=2$						${}_4C_0$	${}_4C_1$	${}_4C_2$	${}_4C_3$	${}_4C_4$	
	$k=3$							${}_4C_0$	${}_4C_1$	${}_4C_2$	${}_4C_3$	...
	$k=4$								${}_4C_0$	${}_4C_1$	${}_4C_2$	...
	$k=5$									${}_4C_0$	${}_4C_1$	...
Total					${}_4C_0$	$\sum_{m=0}^{1} {}_4C_m$	$\sum_{m=0}^{2} {}_4C_m$	$\sum_{m=0}^{3} {}_4C_m$	$\sum_{m=0}^{4} {}_4C_m$	$\sum_{m=0}^{4} {}_4C_m$	$\sum_{m=0}^{4} {}_4C_m$	...
	x_k	...	0	0 ...	0.0625	0.3125	0.6875	0.9374	1.0	1.0	1.0	...

In general, at stage N:

$$M_N = \sum_{k=-N}^{-1} \sum_{m=0}^{k+N} {}_{2N}C_m. \tag{2}$$

The absolute probability, $p_k(N)$ of a particle of component A_1 in address k at stage N corresponds to a linear combination of probabilities of all particles at stage N, i.e.

$$X_k(N) = p_k(N).$$

Note that the probability of each particle being at address k of stage N is independently determined as a coefficient of the binomial series. Also, X_k is the mean concentration of component A_1 in address k at stage N. The concentration distribution at stage N is shown in Fig. 3, where the abscissa is address k, and the ordinate is the concentration of component A_1, X_k.

The fraction of component A_1, α, which remains in and to the right of address $k = 0$ can be expressed by the following equation.

$$\alpha = 1 - \frac{M_N}{M_\infty}$$
$$= 1 - \frac{2M_N}{T_N} \tag{3}$$

where M_N = the number of particles of component A_1 after N stages of transfer to the left of address $k = 0$, which is given by Eq. (2); $M_\infty = \lim_{N\to\infty} M_N$ and T_N = total amount of particles, $2M_\infty$. A plot of ln α vs N in Fig. 4 gives a linear relationship which has been predicted by a deterministic diffusional model[2, also see Sec. III(C)]. From this simple demonstration, we can see that the probabilistic branching model can describe the diffusive mixing mechanism of particles whose prior probabilities are known as the coefficients of the binomial series.

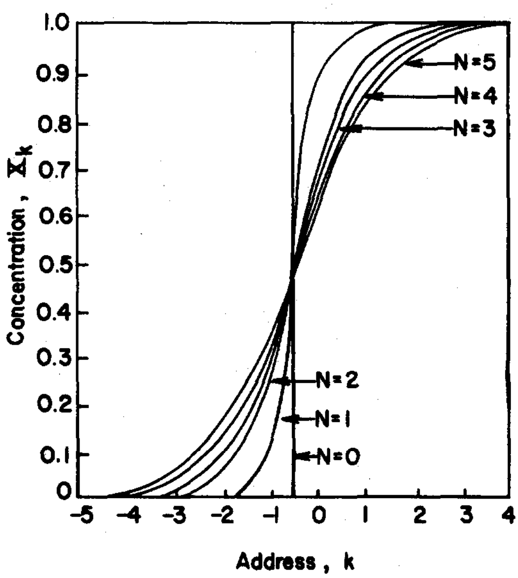

Fig. 3. Concentration profiles as a function of position based on the coefficient of binomial series approach.

(b) *Markov process*

In the probabilistic branching model, each particle is branched to the left or the right with probability of 1/2 at each row. Let Y_i ($i = \ldots, -2, -1, 0, 1, 2, \ldots$) be the random variables which are the new addresses that particles will occupy at each stage. Assume that there is a total of w addresses in the probabiliistic branching model. The event that a particle of component A_1 occupies an address k at stage N is denoted by $Y_N = k$, and the probability that the same particle occupies ad-

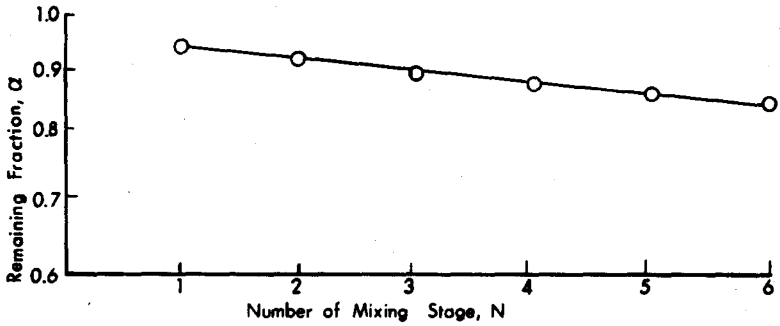

Fig. 4. Variation of remaining fraction vs number of mixing stages.

dress k is represented by $p_k(N)$, i.e.

$$p_k(N) = \text{Prob}\,(Y_N = k). \tag{4}$$

This expresses the absolute probability distribution at stage N.

Let $p_{jk}(N)$ be a transition probability for the moving of a particle from address j to address k with one transition from stage $(N-1)$ to stage N. From the nature of the model, the transition probability is decided by the two events $Y_N = k$ and $Y_{N-1} = j$ only:

$$p_{jk}(N) = \text{Prob}\,(Y_N = k \mid Y_{N-1} = j). \tag{5}$$

Therefore, the mixing process can be regarded as a homogeneous Markov chain. If $p_{kj}(N)$ is considered to be constant at any stage N, i.e.

$$p_{kj}(N) = p_{kj} = \text{constant}, \quad N = 0, 1, 2, \ldots \tag{6}$$

the process is a steady state Markov chain[18]. Since p_{kj}'s are transition probabilities, they must fulfill the following two constraints:

$$0 \le p_{kj} \le 1 \tag{7}$$

and

$$\sum_{j=1}^{w} p_{kj} = 1. \tag{8}$$

For the branching process with equal probability of 1/2 as depicted in Fig. 1, transition probabilities are given and are independent of the position $j (j = 1, 2, \ldots, w)$.

$$\begin{aligned} p_{jj} &= 1/2 \\ p_{j-1,j} &= 1/4 \quad j = 2, 3, \ldots, w-1 \\ p_{j+1,j} &= 1/4. \end{aligned} \tag{9}$$

If the walls of the mixer are the reflecting type for $j = 1$ and $j = w$:

$$p_{11} = p_{ww} = 3/4$$

$$p_{12} = p_{ww-1} = 1/4$$

and

$$p_{ij} = 0 \quad \text{otherwise}.$$

Thus the transition matrix of the process becomes:

$$P = \begin{bmatrix} 0.75 & 0.25 & 0 & \ldots & 0 & 0 \\ 0.25 & 0.50 & 0.25 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & 0.25 & 0.75 \end{bmatrix} \tag{10}$$

From the theories of stochastic processes, absolute probability $p_j(N)$ of a particle in address j at stage N is obtained from the equation [18]

$$\mathbf{p}(N) = p^N \mathbf{p}(0) \tag{11}$$

where:

$$\mathbf{p}(N) = \begin{bmatrix} p_1(N) \\ p_2(N) \\ \vdots \\ p_w(N) \end{bmatrix} \text{ and } \mathbf{p}(0) = \begin{bmatrix} p_1(0) \\ p_2(0) \\ \vdots \\ p_w(0) \end{bmatrix}.$$

Note that $\mathbf{p}(0)$ is the initial distribution of particles. It will be called the "feeding vector". From Eq. (11) and the transition matrix, we can evaluate the probability $\mathbf{p}(N)$ at each address of stage N, and the concentration distribution among all addresses (Fig. 5). Comparison of the calculated concentration distribution (Fig. 5) with that given in Fig. 3, which is based on the calculation of the coefficients of the bionomial series, indicates that the agreement between them is fairly close, since these two approaches are essentially based on the same principles[19]. We can conclude that the degree of mixedness in the horizontal direction at stage N in the branching model mixer, with equal prior transition probabilities, can be predicted from the theories of stochastic processes without experimental measurements of the transition probabilities, as required by other mixers.

(c) *Diffusion theory*

As mentioned in Sec. III(A), particle motions in the probabilistic branching model mixer (Fig. 1) show diffusive mixing in the horizontal direction. The diffusional process can be described by a simple deterministic diffusion equation. Hence, the concentration distribution can be regarded as a function of the continuous variable l which is the horizontal distance from the center line of the mixer. The governing equation for the diffusional process in terms of l and time t is then:

$$\frac{\partial X}{\partial t} = D \frac{\partial^2 X}{\partial l^2} \tag{12}$$

where X is the concentration of the particles of

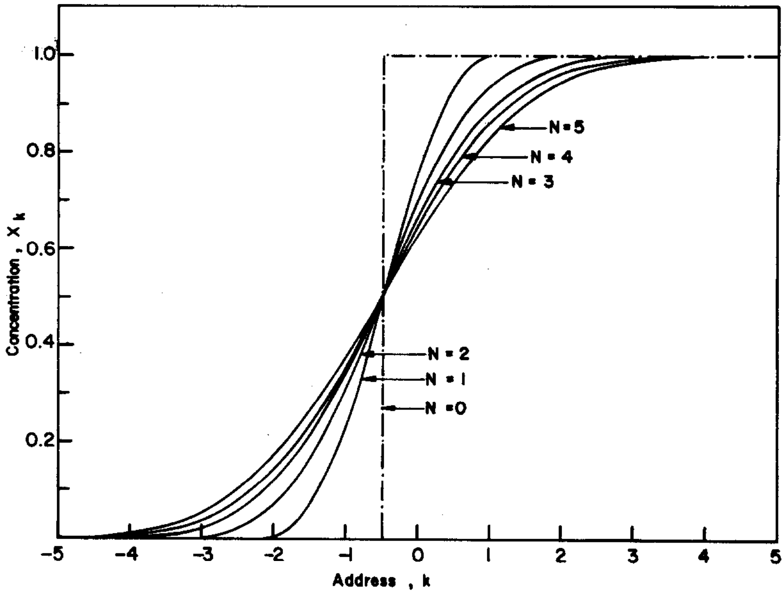

Fig. 5. Concentration profile as a function of position based on the theory of stochastic process.

component A_1 and D is a proportional coefficient term generally called the diffusivity. Since the time of passage for the particles through each stage is approximately equal, t can be replaced by the number of transition stages, N. From the nature of the probabilistic branching model mixer, we can assume the following initial and boundary conditions:

initial conditions:

$$X = 0 \quad -\frac{L}{2} \leq l \leq 0$$

$$X = 1 \quad 0 < l \leq +\frac{L}{2}$$

boundary conditions:

$$\left.\frac{\partial X}{\partial l}\right|_{l=-(L/2)} = 0$$

$$\left.\frac{\partial X}{\partial l}\right|_{l=(L/2)} = 0. \tag{13}$$

The above boundary conditions state that there shall be no net flow of material in the axial direction. The solution to Eq. (12) subject to the above initial and boundary conditions is [20]:

$$X(t, l) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=0}^{\infty} \frac{1}{(2n+1)} \exp\left[-\frac{(2n+1)^2\pi^2 Dt}{L^2}\right] \cdot \sin\left(\frac{(2n+1)\pi l}{L}\right). \tag{14}$$

A plot of concentration X, as a function of distance l/L with (Dt/L^2) as a parameter, is shown in Fig. 6. The average concentration of component A_1 at the left hand side of the center line of the mixer, $-(L/2) \leq l \leq 0$, is

$$X_{[-(L/2),0]} = \frac{2}{L}\int_{-(L/2)}^{0} X \, dl = \frac{1}{2} + \frac{4}{\pi^2} \sum_{n=0}^{\infty} \exp\left[-\frac{(2n+1)^2\pi^2 Dt}{L^2}\right]. \tag{15}$$

When n or D is large, the second and higher order terms in the series at the RHS of the above equation can be neglected. This leads to:

$$\ln\left(\frac{1}{2} - \alpha\right) = A - BN \tag{16}$$

where:

$$\alpha = 1 - X_{[-(L/2),0]}$$

$$A = \ln \frac{4}{\pi^2}$$

$$B = \pi D\tau/L^2$$

$$\tau = \frac{t}{N}.$$

Equation (16) explains the linearity of the plot of $\ln \alpha$ against N shown in Fig. 4.

5. SYNTHESIS OF A MIXING SYSTEM

With an understanding of the two elementary mixing mechanisms, diffusion and convection, a mixer can be synthesized by some combination of the convective model mixer (stratified feeding mixer) and the diffusive model mixer (probabilistic branching model mixer).

Let us now specifically consider a series combination of the two mixers in which the convective model mixer is followed by the diffusive model mixer. In this synthesized mixer, the convective mixing and diffusive mixing occur separately. Let each stria of the stratified feeding mixer contain two columns of identical particles as shown in Fig. 7. The corresponding feeding vector can be expressed as follows:

$$\mathbf{p}(0) = [11001100\ldots1100]^T. \tag{17}$$

If we further assume that the particles fall down independently from the top of the diffusive model mixer (probabilistic branching model mixer) without interaction with other particles, the transition matrix given by Eq.

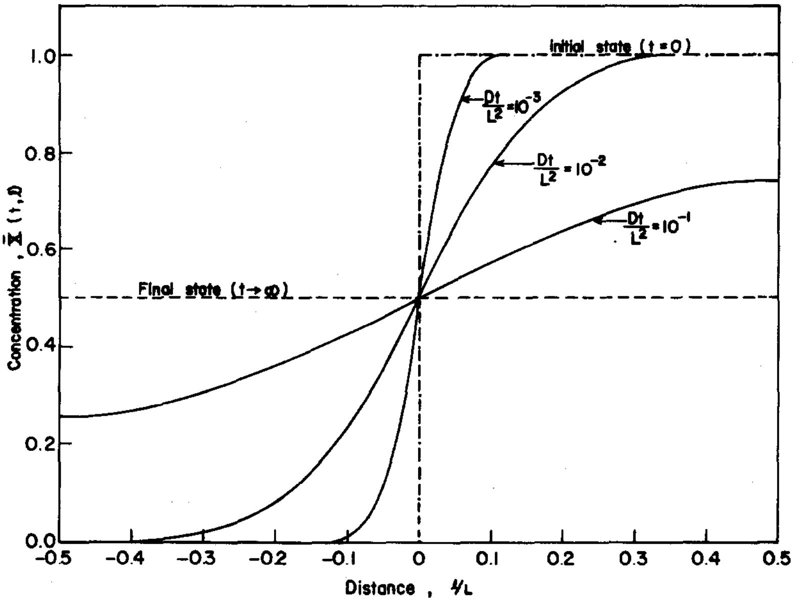

Fig. 6. Distribution of concentration $\bar{X}(t, l)$ vs l/L with Dt/L^2 as a parameter in a two stria feeding vector to the probabilistic branching model mixer.

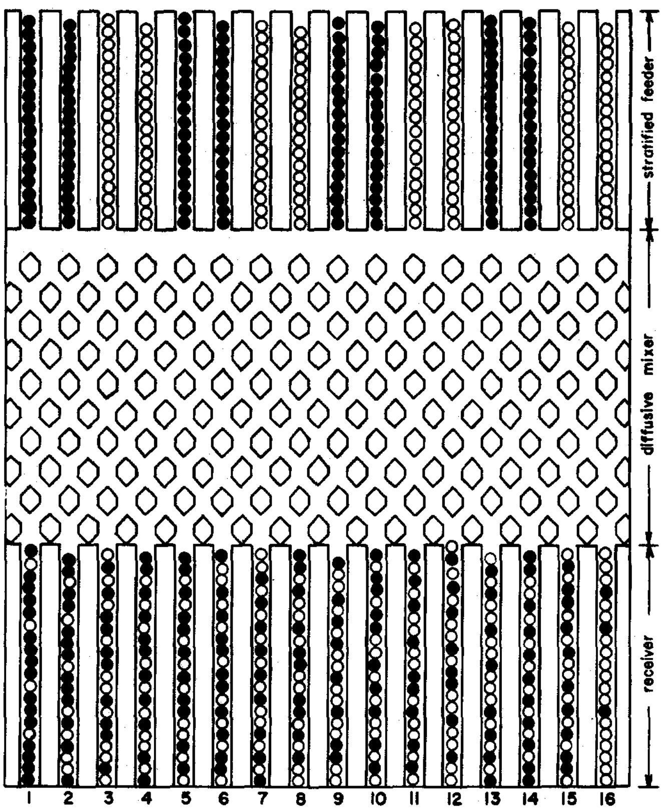

Fig. 7. A. striated feeding mixer and a probabilistic branching model mixer.

(10) is applicable. For this new mixing system, the feeding vector represents the total extent of the mixing due to the convective mechanism, and the transition matrix represents the total extent of the mixing due to the diffusive mechanism. The resulting concentration distribution is shown in Fig. 8. After $N = 5$, the concentration is very close to the equilibrium concentration, i.e. $X_k = 0.5$, $k = \ldots, -2, -1, 0, 1, 2, \ldots$.

Performance of the new mixing system can also be simulated numerically on a computer. Each particle that falls from the top of the system can go either left or right, and random numbers can be generated by simply

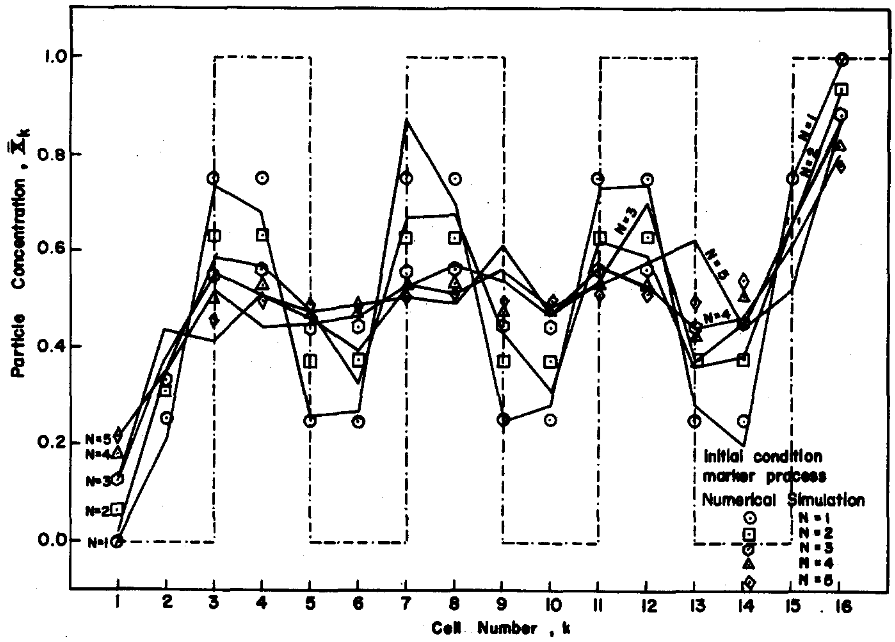

Fig. 8. Concentration distribution in the synthesized mixer with an eight stria feeding vector.

assigning the even numbers to the particles which go to the right and the odd numbers to the left. In other words, a random number is generated to make a decision for a particle encountering a hexagonal block. The result of this simulation is also shown in Fig. 8.

The theory of this approach can be extended to the design and synthesis of other types of mixers. It is not necessary to restrict efforts in synthesis to only the two elementary model mixers; any suitable mixer can be synthesized to form an effective new mixing system. For the case where interactions among particles exist, the above approach still can be applied; however, the transition matrix may become more complex and difficult to obtain. The methods of this study have not actually been applied to the design of industrial solids blending equipment. However, the recent development of a multiport type mixer[21], indicates a possibility of using the methods in practice.

5. EXPERIMENTAL VERIFICATION

Results of simulation presented in the previous section were experimentally verified with the equipment shown in Fig. 9. The hexagonal blocks were fixed on a board made of vinyl chloride plate 3 mm thick. The cells for

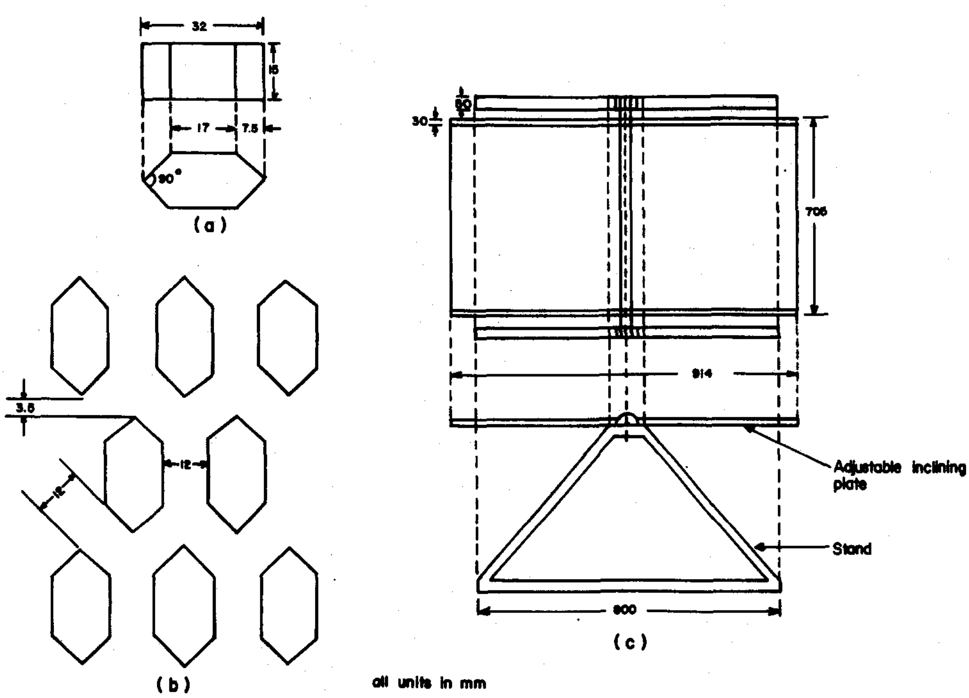

Fig. 9. Experimental set-up.

feeding, which are located at the upper side, and the cells for receiving, which are located at the lower side, are also made of vinyl chloride. The board, initially maintained in a horizontal position, was rotated until the angle of inclination became 12°. The stratified feeding mixer and the probabilistic branching model mixer shown in Fig. 7 are rested on top of the board. This system has 16 cells and five stages, i.e. $N = 5$. Spherical glass particles with a mean diameter of 10.98 mm were used.

First, four particles were allowed to fall from each address at (1, 5, 9, 13). This arrangement was to insure non-interference among the particles. After the first set of four particles at addresses (1, 5, 9, 13) had reached the

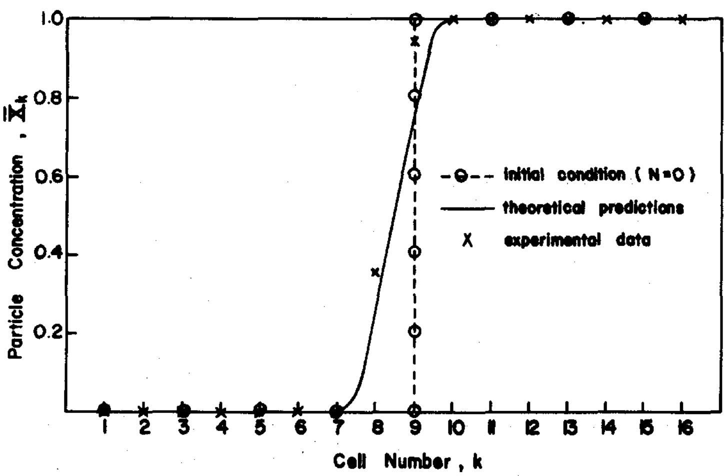

Fig. 10a. Experimental results and theoretical predictions of a two stria feeding vector, $N = 1$.

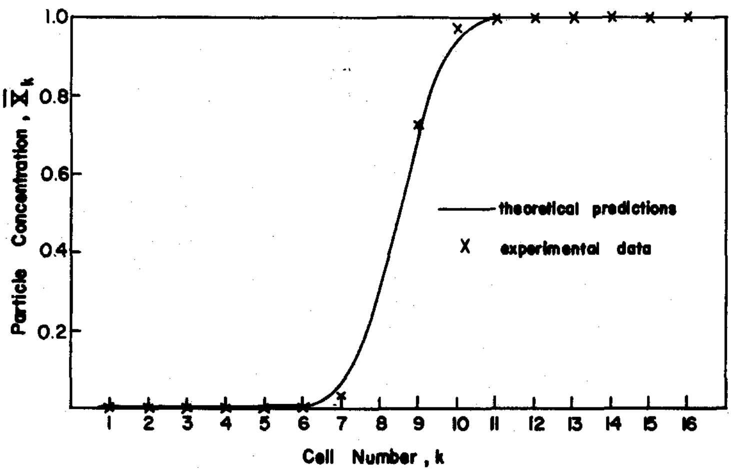

Fig. 10b. Experimental results and theoretical predictions of a two stria feeding vector, $N = 2$.

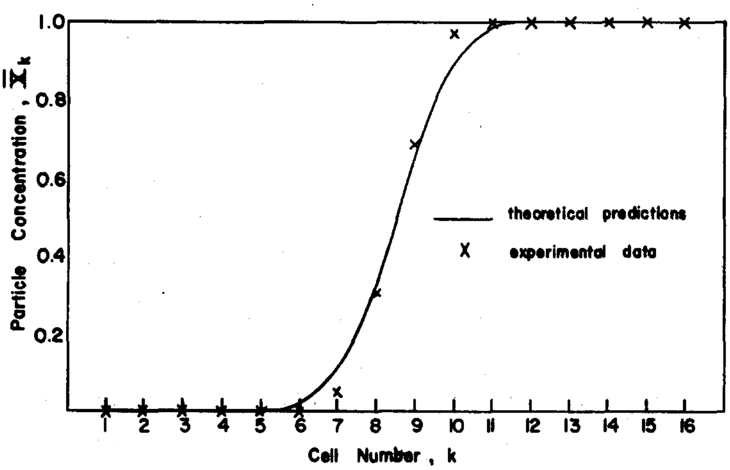

Fig. 10c. Experimental results and theoretical predictions of a two stria feeding vector, $N = 3$.

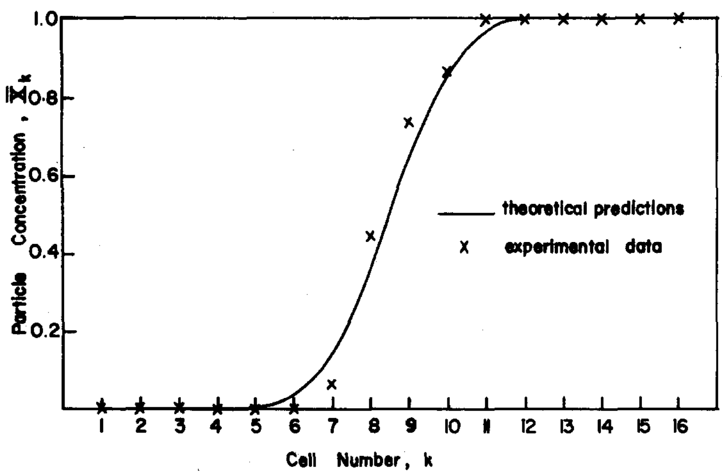

Fig. 10d. Experimental results and theoretical predictions of a two stria feeding vector, $N = 4$.

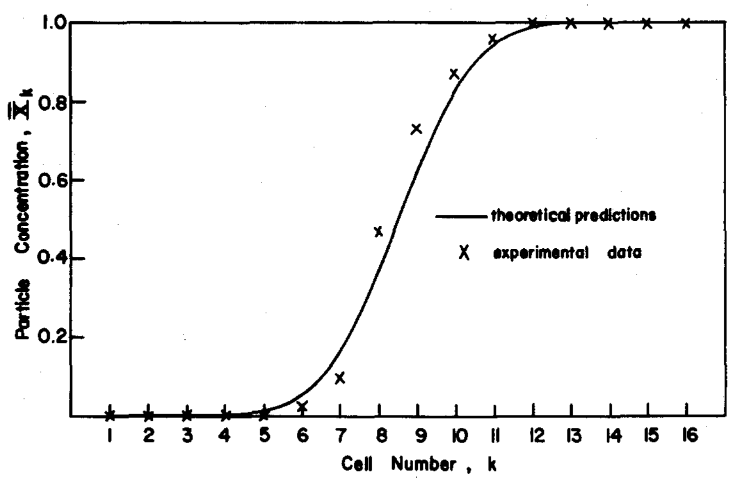

Fig. 10e. Experimental results and theoretical predictions of a two stria feeding vector, $N = 5$.

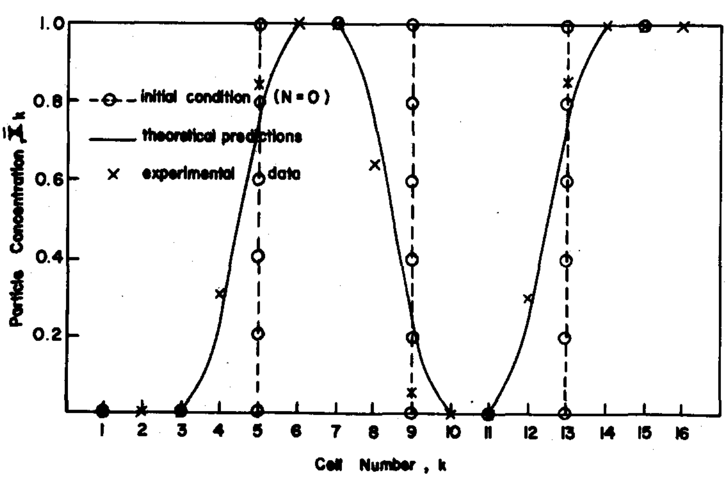

Fig. 11a. Experimental results and theoretical predictions of a four stria feeding vector, $N = 1$.

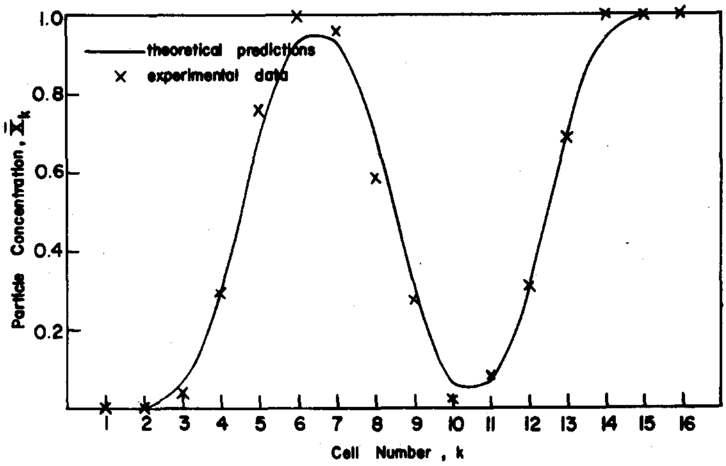

Fig. 11b. Experimental results and theoretical predictions of a four stria feeding vector, $N = 2$.

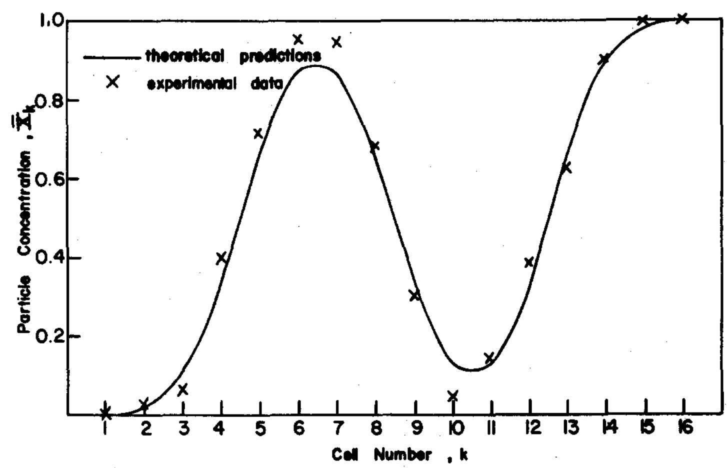

Fig. 11c. Experimental results and theoretical predictions of a four stria feeding vector, $N = 3$.

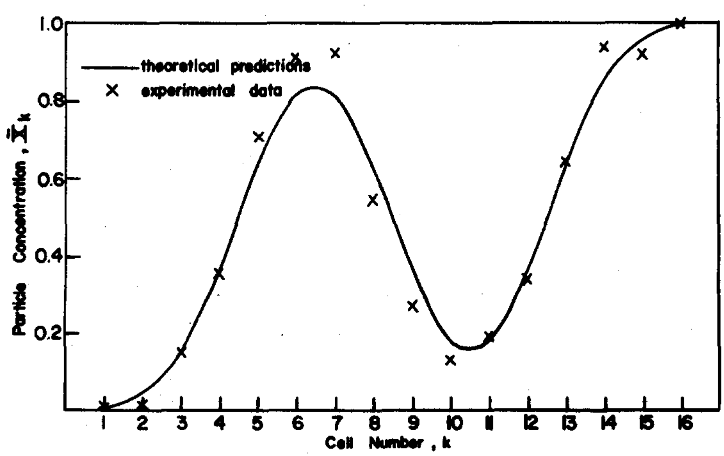

Fig. 11d. Experimental results and theoretical predictions of a four stria feeding vector, $N = 4$.

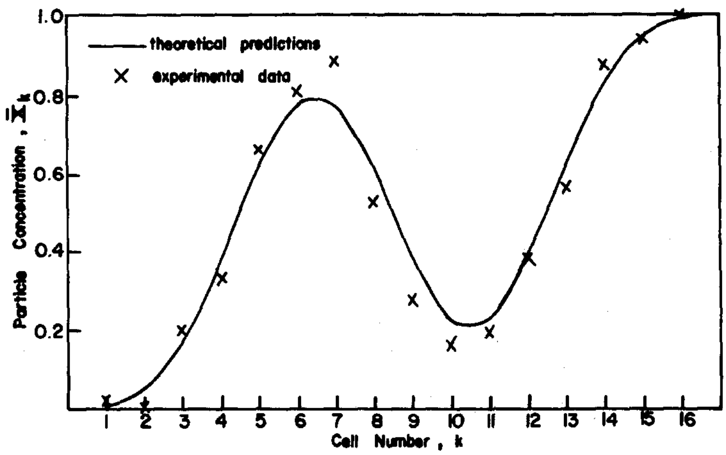

Fig. 11e. Experimental results and theoretical predictions of a four stria feeding vector, $N = 5$.

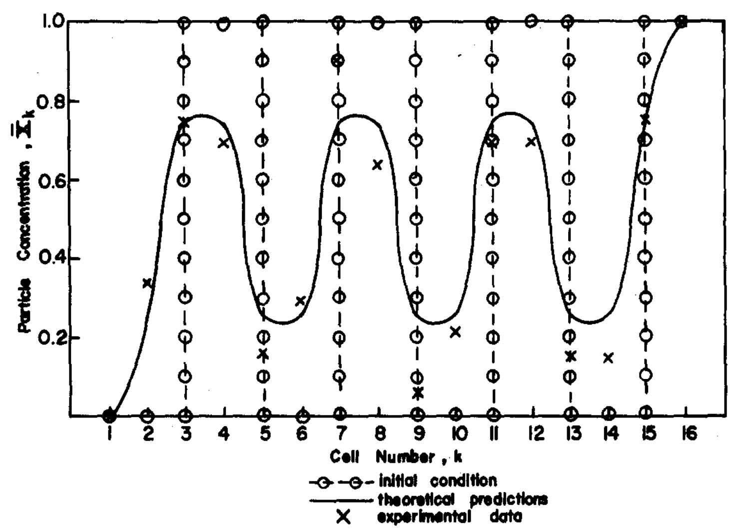

Fig. 12a. Experimental results and theoretical predictions of an eight stria feeding vector, $N = 1$.

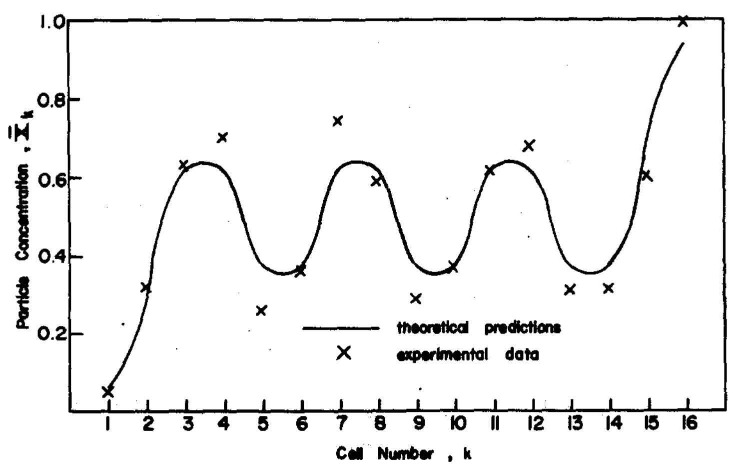

Fig. 12b. Experimental results and theoretical predictions of an eight stria feeding vector, $N = 2$.

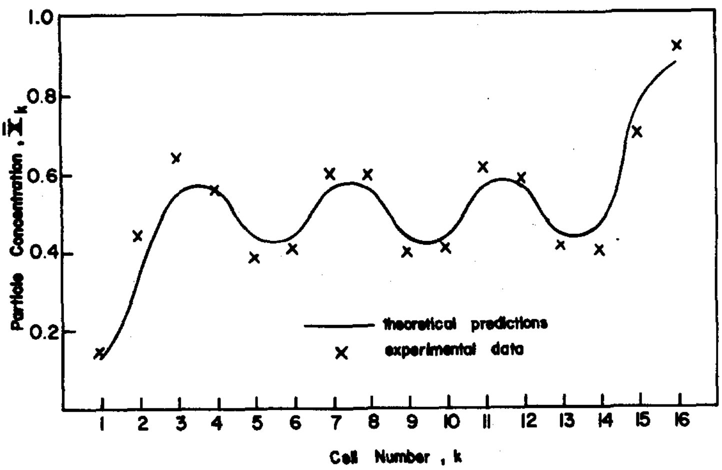

Fig. 12c. Experimental results and theoretical predictions of an eight stria feeding vector, $N = 3$.

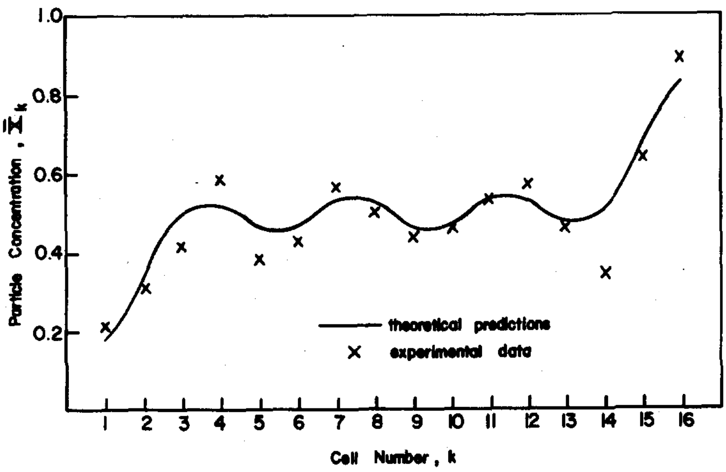

Fig. 12d. Experimental results and theoretical predictions of an eight stria feeding vector, $N = 4$.

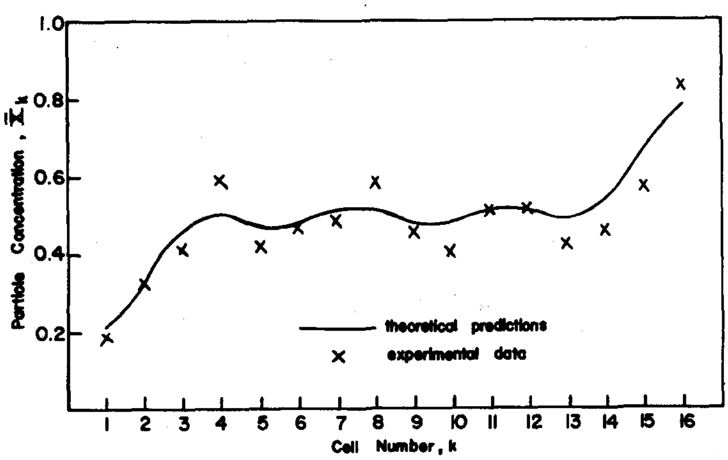

Fig. 12e. Experimental results and theoretical predictions of an eight stria feeding vector, $N = 5$.

second stage, another set of four particles at addresses (2, 6, 10, 14) was allowed to fall. Similarly, the set of particles at addresses (3, 7, 11, 15) was allowed to fall after the set of particles at addresses (2, 6, 10, 14) had reached the second stage, and so forth. This process of feeding was continued for fifty repetitions. The behavior of the particles was recorded continuously by an Ashahi Pentax motor drive camera. In analyzing the photographs, particles that had collided with each other or wedged in the system were eliminated, although there were few such particles. The result at each state N was obtained by counting the number of particles at each address in the receiving columns.

Comparisons of the experimental results with those of theoretical predictions for the case of a two stria feeding vector are shown in Figs. 10(a)–(c). Similar comparisons for four stria and eight stria feeding vectors are respectively given in Figs. 11 and 12. In general, the fluctuations of the experimental data were small.

In summary, the initial concentration distribution was a step function with respect to the spatial coordinate when the particles were completely segregated in the two striae in the feeding section. As the particles passed through an increasing number of stages, N, in the mixing section, this distribution continued to approach a uniform distribution. The increase in the number of striae in the feeding section corresponded in essence to the increased initial extent of mixing. In other words, if the number of striae in the feeding section increases to infinity, the initial concentration distribution would become completely uniform. Experimental results obtained are in good agreement with the theoretical predictions.

In this study, we dealt with the mixing process and stochastic motion of mutually noninteracting particles. The study indicates that further work must be focused on the mixing process of mutually interacting particle systems which is under study by the present authors.

Acknowledgements—The authors wish to acknowledge the U.S. Grain Marketing Research Center, Agricultural Research Service, United States Department of Agriculture and NSF (Grant ENG-73-04008A02) for partial financial support of this research.

REFERENCES

1. P. M. C. Lacey, The mixing of solid particles, *Trans. Instn Chem. Engrs* **21**, 53 (1943).
2. R. Hogg, The mixing of particulate solid materials, Ph.D. Thesis, Univ. of California, Berkeley (1969).
3. Y. Oyama, *Bull. Inst. Phys. Chem. Res.* (Tokyo) **12** (12), Report 1, 953 (1933).
4. P. M. C. Lacey, Developments in the theory of particle mixing, *J. Appl. Chem.* **4**, 257 (1954).
5. S. S. Weidenbaum, A fundamental study of the mixing of particulate solids, Ph.D. Thesis, Columbia Univ. (1953).
6. S. S. Weidenbaum, Mixing of solids, in *Advances in Chemical Engineering*, Vol. II (Eds. T. B. Drew & J. W. Hoopes, Jr.). Academic Press, New York (1958).
7. A. Einstein, *Annln Phys.* **17**, 549 (1905). This and other articles of Einstein have been collected in a book called *Investigations on the Theory of the Brownian Movement* (Ed. R. Fürth, translated by A. D. Cowper). Dover, New York (1956).
8. C. D. Armeniades, W. C. Johnson & T. Raphael (to Arthur D. Little, Inc.) U.S. Patent 3286992 (November 29, 1965).
9. D. S. Pattison, Motionless inline mixers stir up broad interest, *Chem. Engng* **19**, 94 (1969).
10. R. S. Spencer & R. M. Wiley, The mixing of very viscous liquids, *J. Colloid Sci.* **6**, 133–145 (1951).
11. W. D. Mohr, R. L. Saxton & C. H. Jepson, Mixing in laminar-flow systems, *Ind. Engng Chem.* **49**(11), 1855–1856 (1957a).
12. W. D. Mohr, R. L. Saxton & C. H. Jepson, Theory of mixing in the single-screw extruder, *Ind. Engng Chem.* **49**(11), 1857–1862 (1957b).
13. W. D. Mohr, Mixing and dispersing in *Processing of Thermoplastic Materials*, (Ed. E. C. Bernhardt). Reinhold, New York (1959).
14. K. Fushimi, *Probability and Statistics*. Kawade Shobo, Tokyo (1942).
15. Y. Mori, G. Jimbo, M. Taki & F. Washio, Theoretical analysis of mixing mechanism of wetted powders, *Kagaku kogaku* **25**(11), 813 (1961).
16. U. B. Gren, Solids mixing—a review of present theory. *Br. Chem. Engng* **12**(11), 1733 (1967).
17. Y. Inoue & K. Yamaguchi, Particle motion in mixer and mixing process-mixing in a two-dimensional V-type mixer, *Kagaku Kogaku* **33**, 286 (1969) (Japanese).
18. E. Parzen, *Stochastic Processes*, p. 187. Holden-Day, San Francisco (1962).
19. S. Chandrasekhar, Stochastic problems in physics and astronomy, *Rev. Mod. Phys.* **15**(1), 1 (1943).
20. H. S. Carslaw & J. C. Jaeger, *Conduction of Heat in Solids*. Oxford University Press, Oxford (1959).
21. Young Ind., Multi-port gravity blender, Bulletin 842–202, Young Ind., Inc. (1973).